

# Effects of Random Member Length Errors on the Accuracy and Internal Loads of Truss Antennas

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The effects of random member length errors on the surface accuracy, defocus, and residual, internal loads of tetrahedral truss antenna reflectors have been studied analytically. The analytical procedure involves performing multiple, deterministic finite-element structural analyses for a particular truss. For each analysis, the normally distributed, random member length errors are selected by random number generator. A best-fit paraboloid analysis is used to determine a root-mean-square surface error and defocus from each analysis. The statistical properties of these quantities as well as the internal loads are calculated from the results of many independent analyses. Results indicate that the number of members in a tetrahedral truss antenna of a given diameter has a significant effect on surface accuracy, defocus, and internal loads. It was also found that the member axial stiffness and antenna focal length have a very small effect on reflector surface accuracy.

## Nomenclature

$A$	= member cross-sectional area
$\bar{a}$	= arithmetic mean value of the variable $a$
$d$	= length of a core member
$D$	= diameter of the antenna measured between opposite vertices of the hexagonal planform shape
$E$	= member Young's modulus
$f$	= reflector focal length
$\ell$	= nominal length of a surface member
$n$	= number of members in the tetrahedral truss
$N$	= number of trials (number of analyses)
$N_r$	= number of rings in the hexagonal planform tetrahedral truss
$P$	= internal, residual force in a top or bottom surface or core member
$w$	= normal displacement error between a top surface node and the best fit paraboloid
$\gamma$	= ratio of a member's design load to its Euler buckling load
$\Delta f$	= change in focal length from the best-fit paraboloid analysis
$\Delta z$	= translation in the reflector surface in the $z$ direction from the best-fit paraboloid analysis
$\epsilon$	= random member error strain
$\lambda$	= antenna operating wavelength
$\rho$	= member radius of gyration
$\sigma_a$	= standard deviation of the variable $a$

## Subscripts

$i$	= $i$ th truss joint or member
rms	= root-mean-square value

## Introduction

**M**ANY future space missions currently being considered have requirements for large antennas. The reflectors in these antennas range in diameter from 10 to approximately 500 m. The structures that make up these large antennas are likely to have two common features:

1) The antenna reflector support structure will be lightly loaded because external maneuvering and/or stationkeeping forces are usually small.

2) The reflector support structure will need to be extremely accurate to meet the electromagnetic requirement.

A consequence of these two characteristics is that component size variations due to manufacturing or material processing methods may strongly affect the overall shape of the reflector and induce loads in individual components of the structure.

Many different approaches to the design and fabrication of these space-based antennas exist. Usually, the antenna is too large to be transported to space in its working configuration and, therefore, must be deployed or assembled on orbit. This requirement adds to the complexity of the structural design, which, in turn, can introduce many sources for geometric errors in the reflector and the structure supporting the antenna's feed. One reflector concept often considered involves a truss structure that supports the actual reflector surface. This surface could be as simple as flat panels connected directly to the truss joints or a complex network of tension cables supporting a mesh surface above the truss. In both cases, geometric errors in the overall reflector result from errors introduced by the reflector surface itself and from errors introduced by the truss support structure. In many cases, these error sources can be treated independently and then statistically added. This paper considers only the truss support structure.

Double-layered trusses, such as the tetrahedral truss,<sup>1</sup> are an attractive structural concept for antenna reflectors because they combine high stiffness with relatively low weight. It will also be shown that they have an excellent potential for meeting the surface accuracy requirements.

Specifically, the purpose of this study is to assess the effects of random member length errors on the surface accuracy, antenna defocus, and internal, residual forces in tetrahedral truss antenna structures. A similar study was reported in Refs. 2 and 3, which was based on an approximate, continuum analysis of the structure. The study reported herein is based on multiple deterministic structural analyses of the truss using a finite-element analysis. This approach was used successfully in Ref. 4 to assess the effects of individual member length errors on the accuracy of a large, flat microwave power transmission antenna. The present study focuses on obtaining structural accuracy and element force data for a broad class of parabolic truss reflectors which will be compared with results from Refs. 2 and 3.

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## Analytical Procedure

### Tetrahedral Truss Geometry

The geometry of the truss reflectors considered in this study is shown in Fig. 1. The overall reflector has a hexagonal planform and is characterized by the number of rings of members in the truss,  $N_r$ . A typical ring is shown as the shaded region in Fig. 1. The number of rings in the truss is equal to half the number of members along a line between opposite vertices of the hexagon. The diameter of the antenna is given as

$$D = 2N_r \ell$$

The reflector has a paraboloidal shape with a focal length  $f$ . The geometry for the paraboloidal reflector is obtained by first generating joint coordinates for the flat truss based on the number of rings and member lengths. The location of the joints on the concave top surface is then found directly from the equation

$$z = r^2 / 4f$$

where  $x$  and  $y$  are the same as for the flat truss, and  $r^2 = x^2 + y^2$ .

The  $x$ ,  $y$ , and  $z$  locations for the joints on the convex bottom surface are obtained by requiring the lengths,  $d$ , of all members in the core to be exactly equal. In the resulting reflector geometry, the lengths,  $\ell$ , of each of the surface members vary slightly in both the top and bottom surfaces. This condition is defined as the perfect, stress-free state. Deviations in member lengths from this condition cause distortions and residual forces to develop in the structure.

### Member Length Errors

The errors in member length in the truss are assumed to be random and normally distributed with zero mean. It is also assumed that the member length error strain ( $\epsilon_i = \text{error}/\text{member length}$ ) is a general representation of the fabrication errors that could occur in columns of varying lengths. As a consequence of this assumption, longer members have a larger absolute length error than shorter members. Since the mean value of  $\epsilon_i$  is zero, the standard deviation  $\sigma_{\epsilon_i}$  describes the normal distribution.

This error strain measure applies to the members when they are isolated from the truss structure. When these members are force-fit into a structurally redundant truss, forces are developed in the members and overall distortions of the structure result. This same phenomenon would occur in a truss constructed from exact length members but with a random coefficient of thermal expansion under a uniform temperature change.

### Calculation of the Random Response

In Ref. 5, a general, finite-element analysis procedure is described for considering the effects of random element

parameters on structural response. These random parameters could be material properties, cross-sectional dimensions, or member length, etc. The procedure begins by expanding the response quantities in a linear Taylor series as a function of the random parameters. For the case of the truss antenna, the surface error at the  $j$ th node,  $w_j$ , can be expanded as a function of  $\epsilon_i$  as

$$w_j = w_j(\bar{\epsilon}_i) + \sum_{i=1}^n \frac{\partial w_j(\bar{\epsilon}_i)}{\partial \epsilon_i} (\epsilon_i - \bar{\epsilon}_i)$$

where  $w_j(\bar{\epsilon}_i)$  is  $w_j$  evaluated at the mean value of  $\epsilon_i$ , and  $n$  is the number of members. From this equation, the mean and mean-square values of  $w_j$  can be expressed fairly simply. The problem in applying this approach is the high computational expense associated with calculating  $\partial w_j(\bar{\epsilon}_i)/\partial \epsilon_i$  for each  $i$  when the number of members is large. For example, a nine-ring tetrahedral truss has 2160 members.

An alternative approach to determining the statistics of the structural response is by performing a number of deterministic structural analyses with the length error in each member selected by random number generator. This is a traditional Monte Carlo technique. Each analysis in this Monte Carlo approach can be performed for roughly the same computational cost as calculating a set of derivatives  $\partial w_j(\bar{\epsilon}_i)/\partial \epsilon_i$  for a single  $i$ . However, it has been found that reasonable estimates of the statistical averages of the response quantities can be calculated with only 100 to 200 analyses (see Fig. 9), rather than the 2160 derivative calculations that would be required for the nine-ring truss case using the method of Ref. 5. This Monte Carlo approach with the linear finite-element code, EAL,<sup>6,7</sup> is used for all studies reported herein.

### Best-Fit Paraboloid Analysis

The overall quality of the distorted truss surface is assessed by calculating the mean-square error between the resulting reflector structural nodes and an idealized best-fit paraboloid. This best-fit paraboloid is defined as the paraboloid for which the mean-square error is minimum. In determining the best-fit paraboloid, the idealized paraboloidal surface is allowed to translate along the  $z$  axis and rotate about the  $x$  and  $y$  axes relative to the distorted surface defined by the structural nodes. Also, the focal length is allowed to vary. The focal length of the best-fit paraboloid is assumed to be the focal length of the antenna, and the difference between this and the desired focal length is  $\Delta f$ . The translation along the  $z$  axis from the best-fit paraboloid analysis is called  $\Delta z$ . The total axial defocus of the reflector is  $\Delta f + \Delta z$ . This analysis method was taken from Refs. 8 and 9. The root-mean-square normal error,  $w_{\text{rms}}$ , obtained from this analysis is defined as the measure of surface accuracy.

### Nondimensional Statistical Parameters

Displacement errors and member force data were calculated for each trial (analysis). Approximately 100 to 200 trials were run to calculate the statistical properties of this data.

Mean and standard deviation values for the response quantities are calculated from the familiar expressions

$$\bar{a} = \frac{1}{N} \sum_{i=1}^N a_i$$

$$\sigma_a = \left( \sum_{i=1}^N (a_i - \bar{a})^2 / (N-1) \right)^{1/2}$$

where  $N$  is the number of trials,  $\bar{a}$  is the mean value of  $a_i$ , and  $\sigma_a$  is the standard deviation.

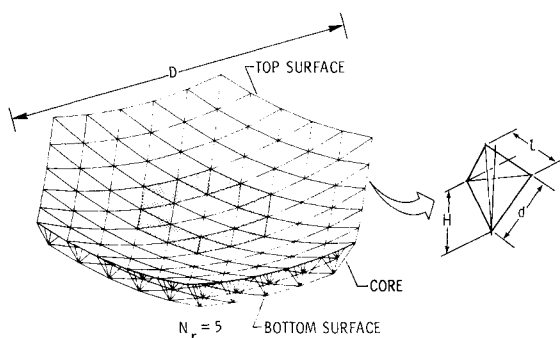


Fig. 1 Parabolic tetrahedral truss reflector structure.

To apply the results of this study to a broad range of sizes and configurations of truss antennas with various member properties, nondimensional parameters involving the mean values and standard deviations of the response quantities are used. The statistical displacement response quantities considered are the mean values and standard deviations of  $w_i$ ,  $w_{rms}$ , and  $\Delta f + \Delta z$ . The quantity  $w_i$  is the normal displacement error at the  $i$ th node in the top surface of the truss. The quantity  $w_{rms}$  is the rms value of all the  $w_i$ , and  $\Delta f + \Delta z$  is the defocus of the antenna. These parameters are nondimensionalized by dividing by  $D\sigma_\epsilon$ . The residual force quantities considered are the mean value of  $P_{rms}$  and the standard deviation of  $P_i$  in the top surface, bottom surface, and core members. The quantity  $P_{rms}$  is the rms value of all the  $P_i$  in either the top or bottom surface or core members. These force parameters are nondimensionalized by dividing by  $EA\sigma_\epsilon$ .

## Results

A number of studies have been performed to assess the effects of member length variations on the surface accuracy, defocus, and internal residual forces of tetrahedral truss antennas. These studies have focused on how various geometric and structural parameters affect truss structural response.

### Surface Accuracy

The data in Fig. 2 show the effect of the number of hexagonal rings in the truss on the surface error parameter. The results are for truss reflectors with an  $f/D=1.0$  and core member lengths nominally equal to surface member lengths. Statistical data were calculated from 100 trials. In addition to the mean value of the surface error parameter, the mean value plus the standard deviation is also plotted to indicate variations among nominally identical truss reflectors. The result from Ref. 2, which is shown as the horizontal line in Fig. 2, is based on a continuum analysis of the truss treated as a free circular plate. As the number of rings increases, the approximation of truss response by a continuum model becomes more valid and the results from Ref. 2 are in better agreement with the finite-element results.

In calculating the surface error parameter  $w_{rms}/D\sigma_\epsilon$ , the spatial distribution of the distortions is lost. However, this spatial distribution is important for two reasons. First, the electromagnetic illumination of the reflector is not uniform, and an accurate assessment of antenna performance requires a consideration of both the illumination pattern and the local surface errors under that illumination. Second, the design and installation of shape correction hardware requires knowledge of both the magnitude and location of the errors to be corrected. Results of a study to address the question of spatial distribution of the local surface errors are shown in Fig. 3.

Figure 3 presents a dimensionless, local error parameter,  $\sigma_{w_i}/D\sigma_\epsilon$ , as a function of the dimensionless distance from the reflector center. The surface error at node  $i$ ,  $w_i$ , has a calculated mean value approximately equal to zero. Thus, the dimensionless standard deviation is plotted in Fig. 3 to characterize the random error. Values of this parameter along each of the six radial lines (see sketch in Fig. 3) are plotted at each radial location. This is shown as a band in the figure. If the standard deviation were calculated from an infinite number of trials, the parameter at each radial location would have a single value instead of a set of six values.

The surface error is fairly uniform in the interior of the antenna for both the five- and nine-ring cases but becomes much larger near the edges. This occurs near the edges because less support of a given member is provided by adjacent members. It is also significant that the nine-ring antenna is more accurate than the five-ring case over the entire surface. This occurs because the nominal member lengths and,

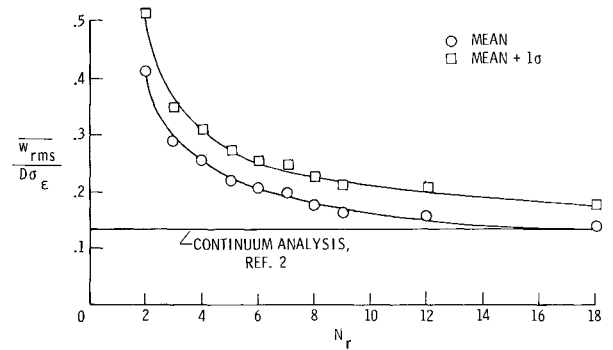


Fig. 2 Effect of number of hexagonal rings on the surface accuracy of a tetrahedral truss reflector ( $f/D=1.0$ ,  $d/l=1.0$ ).

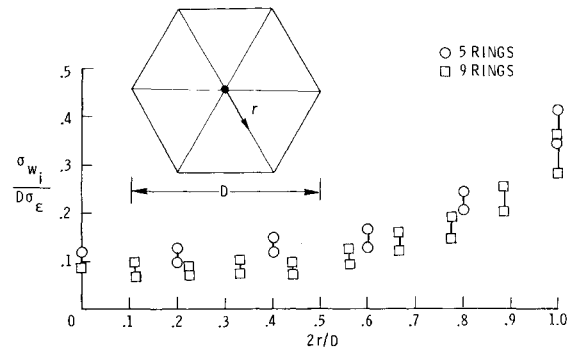


Fig. 3 Spatial distribution of the surface error in a tetrahedral truss reflector ( $f/D=1.0$ ,  $d/l=1.0$ ).

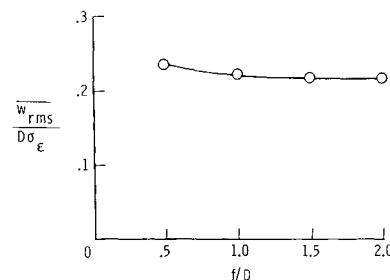


Fig. 4 Effect of  $f/D$  ratio on the surface accuracy of a tetrahedral truss reflector ( $N_r=5$ ,  $d/l=1.0$ ).

therefore, the absolute member errors are smaller for nine rings than for five. Thus, two effects combine to produce the trend shown in Fig. 2. First, the absolute member error decreases with increasing numbers of rings. Second, as the number of rings increases, there are proportionately fewer edge nodes, and their contribution to the overall, averaged surface accuracy decreases.

Antennas with focal length to diameter ratios from about 0.5 to 2.0 are being considered for space applications. Figure 4 shows that in this range,  $f/D$  has a small effect on the truss reflector surface accuracy.

Reference 2, however, shows the effect of  $f/D$  to be greater and in an opposite direction to that shown in Fig. 4. A conclusion of Ref. 2 is that curvature both stiffens the truss structurally and reduces the rms surface error. Figure 4 shows a slight increase in surface error as the curvature increases ( $f/D$  decreases). This increase in surface error results from the assumptions made in generating the truss geometry. For all values of  $f/D$ , the truss had the same planform diameter, and, therefore, the individual member lengths increase slightly as  $f/D$  decreases. Because  $\sigma_\epsilon$  is constant, the absolute member error becomes greater, and this effect offsets any structural benefits of curvature to give a slightly greater rms surface error as  $f/D$  decreases.

One alternative design approach for reducing the surface error of a double-layered truss reflector is to make the core member length different from the nominal length of face members. Figure 5 illustrates the potential of this design approach for five- and nine-ring tetrahedral truss reflectors. For both the five- and nine-ring trusses, there is a small improvement in accuracy over the nominal case ( $d/\ell=1.0$ ) when the core member length is increased to approximately 1.4 times the face member length. The curves show that a significant degradation in the surface accuracy would result if, for other structural reasons, the truss depth were increased to  $d/\ell=3.0$  or larger.

A second alternative design approach for reducing the surface error of a double-layered reflector is making the axial stiffnesses of the different types of members (top or bottom face, core) different. A number of cases were considered for a five-ring truss with  $f/D=1.0$  and  $d/\ell=1.0$  to assess the potential of this. In one study, the axial stiffness of the core members was varied from one order of magnitude less to one order of magnitude greater than that of the face members. In a second study, the bottom face member stiffnesses are varied similarly relative to the core and top face member stiffnesses. In all cases considered, the greatest change in surface error was only 3%. The forces in the members varied greatly from case to case, however. As expected, when a particular set of members was made stiffer, the loads in these members increased.

To determine how errors in the top face, bottom face, and core members contribute as a group to the overall surface error, three cases were considered in which the member length errors were confined to only one of these three groups. For all three cases, the truss had five rings,  $f/D=1.0$ , and  $d/\ell=1.0$ . The values of the surface error parameter for the cases with member errors in the top face, bottom face, or core are 0.124, 0.130, and 0.127, respectively. Thus, there seems to be little advantage to making any one set of members to a higher accuracy standard (i.e., smaller  $\sigma_\epsilon$ ) than the other sets. If these three errors are added, assuming statistical independence, by calculating the square root of the sum of their squares (RSS), the result is 0.220. This agrees with the result from the analysis with errors in all three sets of members.

#### Antenna Defocus

Since the defocus or focal length error of a reflector also affects an antenna's electromagnetic performance, a study was made to determine the effect of random member lengths on this parameter. The dimensionless defocus parameter  $\sigma_{\Delta f + \Delta z}/D\sigma_\epsilon$  is plotted as a function of the number of rings in Fig. 6. All truss reflectors have a nominal  $f/D=1.0$  and  $d/\ell=1.0$ . More trials were required to determine the statistical properties of the defocus parameter with reasonable accuracy than were needed for the surface error parameter. Two hundred trials were used for all but the 12- and 18-ring cases (100 trials for these) in Fig. 6, and the curve is still somewhat rough. The dependency of the defocus error on the number of rings is similar to that of the

surface error; i.e., it appears to be most significantly affected by the reduction in absolute member length error as the number of rings increases. To appreciate the absolute magnitude of the defocus error, consider a five-ring, 50-m-diameter truss antenna constructed from members with  $\sigma_\epsilon = 1.0 \times 10^{-4}$ . Assuming a  $3\sigma$  value of the defocus parameter, the contribution to the defocus error budget from the support structure would be 4.8 cm.

#### Internal Forces

Large space antennas are expected to be designed for very low external loads. Loads that usually have little impact on design, such as residual built-in member forces, can become extremely important and may become a controlling design consideration. Figure 7 shows a statistical member force parameter,  $\bar{P}_{rms}/(EA\sigma_\epsilon)$ , plotted vs the number of tetrahedral truss rings. This parameter is defined such that if the members were completely restrained, the parameter would be approximately equal to 1.0. The values of the parameter in the figure are for members in the top face and core of the truss.  $EA$  values for face members and core members are equal. Results for the bottom face members are nearly identical to those for the top face members. The result from Ref. 3, which is based on a continuum analysis, agrees fairly well with the finite-element data for trusses with a large number of rings.

Figure 8, which is similar to Fig. 3, shows the dimensionless force parameter for the top members as a function of the dimensionless distance from the reflector center. Values of the force parameter along each of the six radial lines from the reflector center to a vertex are plotted as bands in the figure. The radial distance is measured from the reflector center to the center of each truss element along each radial line. For both the five- and nine-ring cases, the member forces are fairly uniform in the reflector interior but drop off significantly for members near the reflector edge. This occurs because there is less restraint provided by adja-

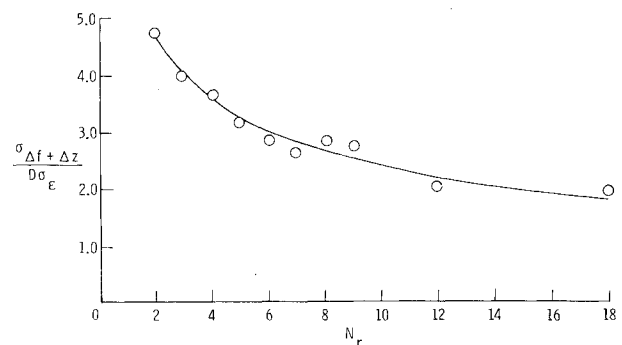


Fig. 6 Effect of number of hexagonal rings on the defocus of a tetrahedral truss reflector ( $f/D=1.0$ ,  $d/\ell=1.0$ ).

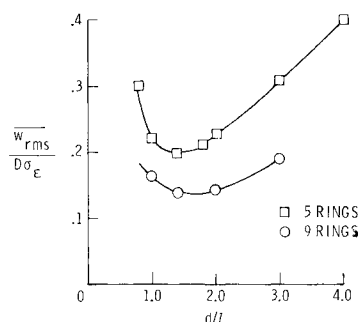


Fig. 5 Effect of core member length on the surface accuracy of a tetrahedral truss reflector ( $f/D=1.0$ ).

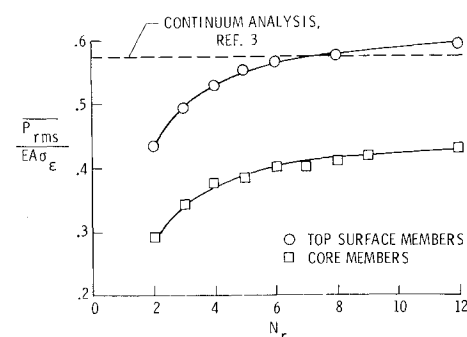


Fig. 7 Effect of the number of hexagonal rings on the residual member forces in a tetrahedral truss reflector ( $f/D=1.0$ ,  $d/\ell=1.0$ ).

cent members near the free edges. The force parameter is defined such that if the members were completely restrained, the parameter would have a value of approximately 1.0. From the figure, it can be seen that the actual restraint provided by adjacent members is significantly less than the fully restrained case. The values of the force parameter for the five- and nine-ring cases are approximately the same in the interior and at the edges. This is consistent with the fact that both the member  $EA$  and  $\sigma_\epsilon$  remain constant as the number of rings is increased.

To investigate whether this internal, residual force is a significant load in the design of the truss member, it is necessary to consider the Euler buckling load of a pin-ended member. This can be written in terms of the member's slenderness ratio,  $\ell/\rho$ , as

$$P_E = \frac{\pi^2 EA}{(\ell/\rho)^2}$$

If the prescribed ratio of member design load to Euler load is denoted by  $\gamma$ , an expression for the required member  $\ell/\rho$  can be written as

$$\left(\frac{\ell}{\rho}\right)_{\text{req}} = \pi \sqrt{\frac{\gamma}{(P_i/EA\sigma_\epsilon)\sigma_\epsilon}}$$

Note that the expression in parentheses on the right-hand side is similar to the statistical member force parameter. Substituting the random parameter  $3\sigma_{P_i}/EA\sigma_\epsilon$  in the above equation gives

$$\left(\frac{\ell}{\rho}\right)_{\text{req}} = \pi \sqrt{\frac{\gamma}{(3\sigma_{P_i}/EA\sigma_\epsilon)\sigma_\epsilon}}$$

where the factor 3 is added to insure a high probability that the random value of the force parameter in any member will be less than the quantity in parenthesis. This is simply the conventional "three-sigma" value. Assuming a value of  $\gamma=0.8$ ,  $\sigma_\epsilon=1.0 \times 10^{-4}$ , and noting from Fig. 8 that the force parameter is approximately 0.63, gives

$$(\ell/\rho)_{\text{req}} = 204$$

Thus, with no external loads on the antenna at all, the members are required to have an  $\ell/\rho < 204$ . If tighter manufacturing tolerances on member lengths can be maintained, and  $\sigma_\epsilon = 1.0 \times 10^{-5}$

$$(\ell/\rho)_{\text{req}} = 646$$

which is a significantly more slender member.

#### Effect of the Number of Trials

A study of the effect of the number of trials on the calculation of the statistical surface error parameter is shown in Fig.

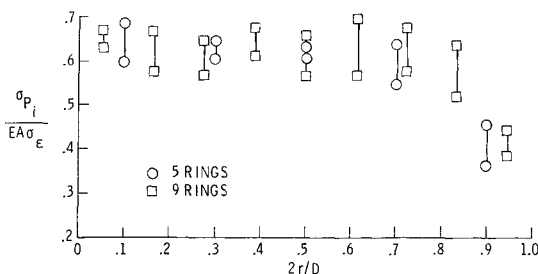


Fig. 8 Spatial distribution of the top surface member forces in a tetrahedral truss reflector ( $f/D=1.0$ ,  $d/\ell=1.0$ ).

9. The mean value and the mean value plus the standard deviation of the surface error parameter are plotted. The reflector is a five-ring truss with an  $f/D=1.0$  and  $d/\ell=1.0$ . The largest difference in the mean value between any two calculated points is about 8.4% and occurs between 10 and 30 trials.

#### Antenna Accuracy Requirements

Figure 10 provides an assessment of achievable surface accuracy for a broad class of antenna reflectors. The surface accuracy requirement for reflectors is often expressed as a fraction of their operating electromagnetic wavelength  $\lambda$ . Thus, the ratio  $\lambda/(w_{\text{rms}} + 3\sigma_{w_{\text{rms}}})$  relates directly to the requirement and is plotted as the ordinate in Fig. 10. The abscissa spans the expected range for  $\sigma_\epsilon$ , assuming careful techniques for manufacturing the truss members.<sup>2</sup> Given the mean value of  $w_{\text{rms}}/D\sigma_\epsilon=0.22$  and  $\sigma_{w_{\text{rms}}}/D\sigma_\epsilon=0.05$ , for a five-ring truss with  $f/D=1.0$  and  $d/\ell=1.0$  (Fig. 2),  $\lambda/(w_{\text{rms}} + 3\sigma_{w_{\text{rms}}})$  can be plotted for reflectors with a broad range of  $D/\lambda$  values.

With a specified rms surface error requirement for the reflector support structure and a  $D/\lambda$  for a particular antenna, Fig. 10 can be used to determine the required member accuracy. For example, consider an rms surface error requirement of  $\lambda/50$  for the land mobile satellite that has a  $D/\lambda$  of about 480. The intersection between a horizontal line through the ordinate value of 50 and the  $D/\lambda$  line for the land mobile satellite has an abscissa of approximately  $1.5 \times 10^{-5}$ . This is the required  $\sigma_\epsilon$  for the truss members.

Values of  $D/\lambda$  in the range 100 to 1000 are typical of communications antennas such as the land mobile satellite.<sup>10</sup> The microwave radiometer spacecraft considered in Ref. 11 has a

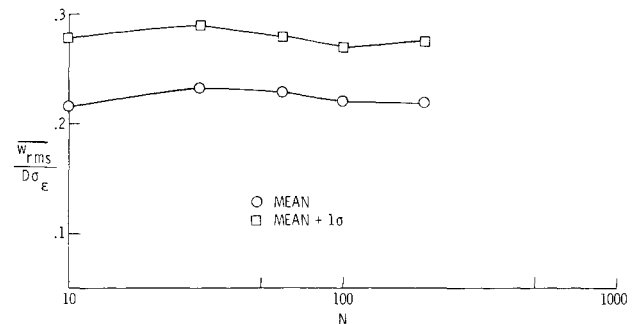


Fig. 9 Effect of number of trials on the calculation of statistical properties of the reflector surface error ( $f/D=1.0$ ,  $d/\ell=1.0$ ,  $N_r=5$ ).

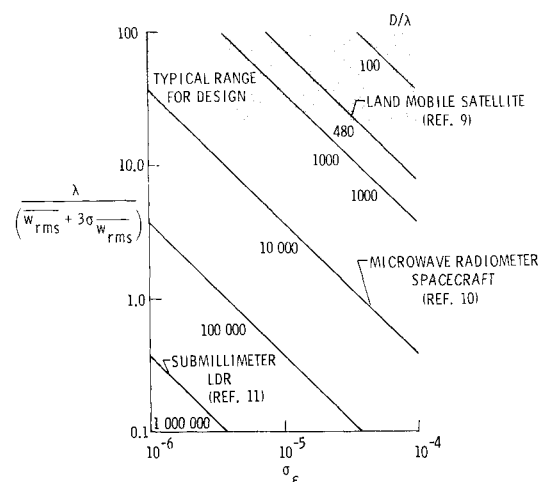


Fig. 10 Achievable values of surface accuracy with a five-ring tetrahedral truss for different classes of antenna reflectors.

$D/\lambda$  of approximately  $1 \times 10^4$  and the infrared wavelength large deployable reflector (LDR) concept of Ref. 12 has a  $D/\lambda$  of about  $1 \times 10^6$ . Typical rms surface accuracy requirements range from  $\lambda/20$  to  $\lambda/100$ .

Whether the surface accuracy requirement for a particular mission can be met with tetrahedral truss support structure depends on the proportion of the total reflector error budget allocated to the truss. However, it can be seen from Fig. 10 that the accuracy requirements for reflectors with  $D/\lambda$ 's in the 100 to 500 range can probably be met with tetrahedral truss support structure. For very short wavelength reflectors such as LDR, the error in the support structure alone exceeds the total error budget for the reflector by a significant margin.

### Concluding Remarks

An approach for determining the effects of random member length errors on truss reflectors and results from a number of parameter studies have been presented. These results enable a designer to assess the potential of tetrahedral truss support structure for meeting accuracy requirements (both surface and defocus) for a broad range of parabolic reflectors. It is also possible to estimate residual member loads due to random member length errors. Because other operational loads are low, this loading condition may be dominant.

From the results of the studies presented, the following conclusions can be drawn:

1) Increasing the number of rings in the truss substantially decreases the relative surface error and antenna defocus, and increases the residual member forces. The surface error and defocus are reduced primarily because the absolute member error decreases as the number of rings increases. Some of the error reduction is also due to the greater percentage of "interior" nodes in the surface relative to "edge" nodes. The restraint provided by adjacent members in the interior of the antenna reduces the nodal displacements and increases the member forces.

2) In the range of focal length to diameter ratio of 0.5 to 2.0,  $f/D$  has a small effect on surface accuracy.

3) Making core members slightly longer than face members improves surface accuracy only slightly. However, if the core members are shortened or lengthened significantly relative to the face members, the surface error increases rapidly.

4) Having different structural stiffnesses for the top and bottom face members or core members offers little potential for surface accuracy improvement.

5) The contributions of the individual member errors from the face and core member to the overall surface error are approximately equal. Thus, surface accuracy cannot be greatly improved by tightening manufacturing tolerances on a single group of members.

6) For truss antennas with a large number of rings, surface accuracy and residual force data from a continuum analysis agree reasonably well with finite-element results.

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